THE NORMAL, THE FAT-TAILED, AND THE CONTAGIOUS: MODELING CHANGES IN EMERGING MARKET BOND SPREADS WITH ENDOGENOUS LIQUIDITY*

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Abstract

We simulate an agent-based model in an attempt to replicate the statistical properties of daily changes in emerging market bond spreads over US treasuries, namely: 1) excess kurtosis; 2) serial correlation, suggesting deviation from market efficiency; and 3) excessive co-movement, suggesting contagion. A simple model of interacting traders produces alternating booms and crashes, as in reality, but is not capable of producing fat-tailed distributions or contagion. An extended model with market makers whose bid-ask spreads widen with increased volatility and whose prices depend on the size of their inventory does produce excess kurtosis and correlated spreads, even though the fundamentals of the two countries are independent. This model highlights the role of liquidity (or lack of it) in explaining large rate movements and contagion.

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I. Introduction

Despite extensive study of capital flows to developing countries, it is safe to assert that there remain a number of issues about which there is not yet a clear consensus among economists[1]. Three important issues stand out. First, though there are many models of balance of payments crises, there is little agreement in particular cases on the dominant cause. Specifically, are crises the results of poor economic fundamentals or of a self-fulfilling crisis triggered by a rush for the exits by investors?[2] Are the relevant economic fundamentals excessive monetary expansion, government deficits, or financial sector problems? Second, does a crisis in one country trigger one in another? As is the case for a financial crisis in a single country, here also it is necessary to identify the relevant set of fundamentals, since there are a number of linkages that would explain the co-movement in financial variables across countries. This leads to the third issue: what are the causes of co-movement: macroeconomic fundamentals, or instead financial contagion operating through various channels, including shifts in investor attitudes, balance sheet effects, or regional portfolio rebalancing triggered by a crisis in one country? In other words, is co-movement excessive?

In this paper, we do not test statistically among the various channels that have been advanced in the literature, but rather simulate a simple model of balance of payments crises, focusing on both interacting expectations and varying liquidity as important features affecting emerging market bonds. We posit a simple model of currency crises (which admits of both fundamental and self-fulfilling triggers of crises) where investors hold both emerging-market and developed country assets. Thus, portfolio rebalancing could, in principle, lead to co-movement in asset prices. We depart from the assumption made in most models of contagion, namely the assumption that there is a representative agent forming rational expectations. Instead, investors form expectations on the basis both of their past experience and of imitation of other (more successful) investors. These assumptions are sufficient to produce interesting dynamics, quite independent of the fundamentals, and we study their implications for triggering crises and causing contagion. This work is in the tradition of (multi-)agent-based models, which have been widely applied in a number of disciplines; some of that work is briefly surveyed below.

Our aim is also to see whether such a model can replicate in more detail the properties of the distributions of interest rates on emerging-market bonds. While there is a considerable literature on the structure of returns in various developed-country financial markets (e.g. Mandelbrot (1963), Mantegna and Stanley (1995), and Bouchaud and Potters (2000)), emerging market bonds have received less attention. In a companion paper, Masson (2003), daily data for JP Morgan's Emerging Market Bond Indices Global (EMBIG) were analyzed for the 20 or so countries where data were available. The following properties were identified on the basis of statistical analysis of the distributions of daily changes in spreads relative to US treasury securities. **First**, the distribution is very much not normally distributed, but rather exhibits fat tails indicating that extreme events are much more likely than for the normal. This is a stylized fact that is significant, because it may allow one to distinguish among models of financial crises

[1] A sampling of the recent literature is contained in Claessens and Forbes, eds. (2001).

[2] See for instance, the debate between Obstfeld on the one hand and Krugman and Garber, on the other, about the causes of the Mexican crisis of 1994-95 (see Krugman, 1996). Krugman, however, changed his position following the Asian crisis, now admitting that there were self-fulfilling elements.

and contagion. **Second,** changes in spreads are serially correlated, indicating a possible departure from market efficiency. This applies to almost all emerging market bonds, and typically the first-order serial correlation coefficient is positive, and significant. **Third**, there is evidence of contagion, defined as excessive co-movement: changes in spreads (and hence asset returns) are considerably more correlated across countries than are macro-economic fundamentals defined to include trade between countries.

Agent-based computational approaches have advanced understanding of what causes heavy-tailed price movements. Lux and Marchesi (1998) demonstrated that multi-agent models can produce heavy-tailed return distributions without assuming similarly distributed movements in fundamentals. Farmer and Joshi (2002) showed that market structure plays a large part in determining market behavior, and Farmer et al. (2004) that liquidity can drive the production of heavy-tailed price movements. MacKenzie (2003) investigated empirically the social structure of emerging market investment decisions, providing support for the theory that imitation may be a major mechanism in producing heavy tailed returns and excessive co-movement between markets with uncorrelated fundamentals.

While this area of research is often associated with collaborations between economists and physicists, evolutionary biology has become increasingly influential in understanding market dynamics. Gandolfi et al. (2002) provides a survey of these linkages. Farmer (2002) demonstrates that useful parallels can be drawn between multi-agent financial market models and standard models in population biology, where survival depends on "fitness." Blume and Easley (2002) examine fitness dynamics in a market setting, demonstrating that market selection favors profit maximizing firms, but leads to systems that are much less stable than standard profit maximizing models would predict.

In this paper, we simulate various parameterizations of a model of investment in emerging markets in order to see what features are necessary to replicate these stylized facts. This model is an extension of the single-country, multi-agent model in Arifovic and Masson (2004). It combines a simple balance of payments crisis model with hypotheses concerning the formation of expectations by investors. In particular, we assume that investors form and update their expectations on the basis of the success (or otherwise) of their investment strategies. If the latter are successful (in the sense of giving a better return than some randomly chosen comparator), then the investor retains the expectation and strategy; otherwise, the investor adopts the comparator's strategy. In addition, investors at times experiment by randomly choosing a new rule. Thus, the investors in the model do not have information about economic fundamentals (the evolution of the trade balance and foreign exchange reserves), but adapt their strategies on the basis of past results. This has the potential of producing bandwagon effects, that is, serial correlation of changes of asset holdings and spreads, as expectations of excess returns for a particular asset become self-fulfilling, reinforcing that asset's attractiveness. Finally, both economic fundamentals and investor behavior contribute to triggering crises, which occur when a country's reserves go to zero, forcing it to default: this can occur as a result of a bad shock to the trade balance or because a sufficient number of investors withdraw their capital (a "sudden stop" in Calvo's words[3]). The model, when implemented for a single emerging

[3] See Calvo (1998).

market bond, has some success in producing an alternation of booms and crashes in emerging markets, similar to actual data (see Arifovic and Masson (2004)).

The model also succeeds in producing serial correlation of changes in spreads, as is present in the actual data. It is clear that the "bounded rationality" of investors means that past success in investing reinforces strategies in a way that produces serially-correlated changes in returns. To the extent that imitation occurs also, there will be herding, i.e. reinforcement of strategies **across** investors. In Arifovic and Masson (2004), that herding is the cause of booms and crashes in investment in emerging markets.

However, the model in Arifovic and Masson (2004) does not include more than one emerging market bond, and several are needed to model contagion across markets. Moreover, that model assumes that all liabilities are short-term, so that the liquidity of a secondary market does not come into play. It turns out that a simple extension of the model to two emerging market bonds with market clearing prices is not able to replicate two of the stylized facts mentioned above: the basic model could not produce fat-tailed distributions, nor did co-movement in interest rate spreads for pairs of emerging market countries emerge when the fundamentals were not themselves correlated. However, an extension of the model to include lack of liquidity is able to reproduce these two properties of the actual data, excess kurtosis and excessive co-movements across countries. We present below simulations of various parameterizations of the basic model without varying liquidity; while the first two moments of the distribution can easily be reproduced, the fourth moment is much smaller than in the real-world data, indicating that the simulated distribution has much thinner tails than the actual one. Indeed, simulated changes are even more thin-tailed than the normal. Furthermore, correlations in emerging market spreads are small, even when the fundamentals are assumed to be highly correlated. Instead, the model would predict small negative correlations, as portfolio shifts out of one asset, would, other things equal, produce inflows into the others. It is clear that herding behavior, which is consistent with positive serial correlation in returns on individual country bonds, need not produce contagion, which requires some cross-country linkage based on economic fundamentals, correlated expectations, shifts in attitudes to risk, or portfolio rebalancing affecting the whole asset class.

The complete model includes a market in emerging market bonds in which liquidity is provided by a market maker. Following the literature, market makers' bid-ask spreads are assumed to vary positively with the volatility of asset prices, not only in that security but also others, while the market maker's mid-market price varies inversely with the size of the inventory held of that security. The changing degree of liquidity in the market provides a possible explanation of extreme movements of interest rates. Market practitioners point to the fact that at times of crisis, the market "dries up", as everyone attempts to get out at the same time. Moreover, market makers, who typically deal in a number of different securities, react to losses in one market by increasing their bid-ask spreads for the other bonds in which they deal. Thus, lack of liquidity may spill over to other emerging market bonds.

This version of the model is capable of producing the excessive kurtosis and excess co-movement that is present in the actual data. While it is not the only possible explanation—and we discuss an alternative version of our model that is capable of producing the excess kurtosis and contagion—it explores a plausible channel that has so far received little attention in the literature. We think that it deserves further exploration, both on the side of modeling as well as detailed study of the way the trading in emerging market bonds works. The microstructure of trading in emerging market bonds seems to be a promising area for future research, and one that has so far received little attention, unlike the foreign exchange market (see, e.g., Evans and Lyons (2002)).

The plan of the paper is as follows. The next section describes the basic model, which draws on Arifovic and Masson (2004). Section III details the statistical properties of the distribution of simulated emerging-market spreads using various parameterizations of the model with a single emerging market bond; for none of them does excess kurtosis emerge. Section IV simulates the same model with two emerging market bonds, noting that despite herding behavior (resulting from investors imitating other successful investors), there is no correlation of spreads across countries. Section V introduces a more general model with two-period emerging market bonds, which can be traded in the period before they mature. This model is shown, when liquidity as provided by a market maker is endogenous, to produce excess kurtosis and excessive co-movement. Section VI concludes.

II. A Model of Emerging Market Crises

We proceed to describe a canonical balance of payments crisis model, and in the next section examine to what extent it can replicate the actual data. In this model, all capital flows are assumed to take the form of purchases or sales of the debt of the emerging market government. The model links the ability of a country to service its debts to the existence of non-negative reserves: once reserves hit zero, a default is triggered, leading to losses by investors. The evolution of the balance of payments, i.e. the sum of the trade balance, minus interest payments abroad, plus net capital inflows, is the key to the ability of a country to repay its borrowings, and the interest on them.

Foreign investors choose from a very simple menu of investments: in the one-emerging-market case (to be generalized to several below), they form expectations of the probability of a default on emerging market debt, and choose to invest either in the safe (US treasury) security, paying a known and constant return r^*, or the emerging market bond, paying r_t. The amount that they invest this period in the emerging market bond, summed across all investors, is denoted D_t.

A default occurs at *t* if reserves would have gone negative. The basic balance of payments equation in the model is:

$$R_t = R_{t-1} + D_t - (1 + r_{t-1})D_{t-1} + T_t \qquad (1)$$

where R_t are reserves, and T_t is the trade balance[4]. The trade balance is a stochastic process which in this model constitutes the economic fundamental.

Investors form expectations of the probability of default. Let investor i's estimated probability be π_t^i and expected size of the default be δ_t^i. We assume that the market interest rate is set to equal to the US rate plus the average of all *n* investors' expectations[5]. More exactly, the market rate plus unity is a geometric average over unity plus the expected probability times the size of devaluation, times unity plus the US rate:

$$1 + r_t = (1 + r^*)\left(\prod_{i=1}^{n}(1 + \pi_t^i \delta_t^i)\right)^{1/n} \qquad (2)$$

This formulation allows us to determine both the interest rate, which reflects average expectations, and the quantity of capital flowing to emerging markets, which reflects the skewness of the expectations of default. To illustrate this assume that risk aversion is zero, so that an investor puts all her money in either the safe asset or the emerging market bond, whichever pays the higher expected return. If an investor then has a more optimistic assessment of the probability (and size) of default than the average embodied in r_t, she will put all her money in emerging market bonds (negative holdings of either asset are ruled out). If less optimistic, then she will put her wealth into the safe asset. In these circumstances, the skewness of the distribution of expectations across investors will determine the amount that is invested in emerging markets: positive skewness of the distribution of devaluation expectations will indicate that more than half of investors are to the left of the average (hence more optimistic) so that investment in emerging markets will be higher than in the case of negative skewness (see Arifovic and Masson (2004)).

Investors are assumed not to observe the economic fundamental (the trade balance) or reserves. While an extreme assumption, it reflects a reality noted by observers of this market, namely the ignorance of many investors in emerging market bonds (who then "woke up" to the flaws of the Asian tiger economies after the crises occurred—see Goldstein (1998)). Starting from some distribution of initial priors, expectations are updated on the basis of past investment returns, with an element of imitation and experimentation. In particular, if investor i puts a proportion x_t^i into the emerging market bond (and the rest into the safe asset), by analogy with evolutionary biology (Blume and Easley (2002), Gandolfi et al. (2002), Bowles and Hammerstein (2003)) one can define "fitness" as

$$\mu_t^i \equiv (1 - x_t^i)(1 + r^*) + x_t^i(1 + r_t)/(1 + \delta_t) - 1$$

where δ_t is the **actual** default size (or zero, if no default) in period t. The variable μ_t^i is in fact the realized rate of return on investor *i's* portfolio. Each investor is assumed to observe the

[4] Reserves could also be assumed to earn interest, but since the US rate is assumed constant this complication adds little to the model.

[5] The basic model thus assumes risk neutrality. When investors are risk averse, a risk premium that reflects the average across investors is embodied in the market return on the emerging market bond.

expectations and fitness of another investor, chosen at random[6]. Investor i, in updating her expected probability and size of default (π_t^i, δ_t^i), will compare the fitness of her expectations with those of a randomly chosen comparator (where the probability of being picked depends on relative fitness—i.e., more successful rules are more likely to be imitated[7]); if the latter's fitness is greater, she will adopt the comparator's expectations; if less than or equal, keep her own. In addition, with some probability p_{ex} she would simply discard her expected probability of default π_t^i and pick a new one randomly, drawn from a uniform distribution on the interval $[0, \pi^{\max}]$, and similarly for the size of default, if it is endogenous. However, in the simulations below we assume for simplicity that the size of default is fixed and known, because, for instance, a default triggers fixed costs that are independent of the amount of the shortfall of reserves. So, in this case, both expected and actual default size (if one occurs) are known and equal to $\bar{\delta}$. We will henceforth assume this to be the case.

Investors' wealth is endogenous and evolves over time, depending on investor strategy and the rate of return:

$$W_t^i = (1+\mu_{t-1}^i)W_{t-1}^i - \bar{r}W_{t-1}^i \qquad (3)$$

where the last term is consumption out of wealth (at a constant, exogenous rate $\bar{r}$). The model is completed by a stochastic process for the trade balance. This specifies the trade balance as an AR(1) model:

$$T_t = \alpha + \beta T_{t-1} + u_t \qquad (4)$$

where $u_t \sim N(0,\sigma^2)$. Estimates based on annual data for various countries are found in Masson (1999a). An isomorphic model would replace the balance of payments equation by the government's budget constraint, impose an upper bound on debt (provoking default if reached), and replace the trade equation with a stochastic process on the primary (non-interest) government deficit. Such a model would give qualitatively similar results.

The simplest version of the model is as described above. However, there are two further complications that need to be explained: 1) portfolio selection when investors are risk averse, and 2) conversion of the model from an annual frequency to the monthly or daily frequency that matches our empirical data for emerging market spreads. These complications are briefly discussed here; details are given in appendices.

To account for risk aversion, we assume that investors maximize expected utility. Substituting into the first order conditions a second-order Taylor's expansion of the utility function, we obtain the familiar mean-variance model of choice between a riskless asset and one or several risky assets. For the case of just one emerging market bond the resulting expression for the proportion of the portfolio held in the emerging market bond will be given by

[6] Other information structures are of course possible; for instance, only particularly visible investors (like hedge funds) known to have expertise could serve as comparators; we discuss this possibility in the concluding section.
[7] As in Arifovic and Masson (2004), if returns are negative, the underlying expectations are not imitated.

$$x_t^i = \frac{b^i (r_t - \frac{1+r_t}{1+\bar{\delta}} \pi_t^i \bar{\delta} - r^*)}{\pi_t^i (1-\pi_t^i) \bar{\delta}^2} \quad (5)$$

The expression in the denominator is the variance of the return on the risky asset, while the numerator (multiplied by a parameter b^i that is the inverse of the coefficient of risk aversion) is the expected yield differential in favor of the risky asset, if positive.

If $0 < x_t^i < 1$, then the proportion accounted for by the emerging market bond in *i's* portfolio given by (5): if not, then $x_t^i = 0$ or $x_t^i = 1$. In the limiting case of zero risk aversion ($b^i \to \infty$), investors merely select the asset yielding the highest expected return (given the constraints $0 \le x_t^i \le 1$). The general case is discussed in Appendix I.

For the model to be useful it needs to integrate high frequency financial markets with lower frequency economic fundamentals. Appendix II discusses the approach taken, namely to convert equation (4) to a monthly or daily autoregression on the assumption that the true stochastic process in fact operates at the higher frequency. In addition, adjustments have to be made to make stocks and flows consistent. Interest rates have to be scaled appropriately, as does the probability of default. At a daily frequency, it makes little sense to update expectations on the basis of one-period returns; thus, we specify a memory horizon $h \ge 1$ over which past returns are averaged when comparing fitness with a comparator[8]. Finally, we take account of the possibility that not all investors are active at a daily frequency; we specify a probability p_{inv} that an investor will update expectations and alter her portfolio in any given period.

III. Results of a Simple Model with One Emerging Market Bond and a Safe Asset

The model is essentially that of Arifovic and Masson (2004), calibrated to the reserves and external debt of Argentina in 1996, and using a stochastic equation for the trade balance that is estimated with historical data[9]. As in Arifovic and Masson, the model produces a succession of booms and crashes. However, as we will see it does not produce a distribution for the changes in spreads that has fat tails.

The model was first converted to a daily frequency, using the method described in Appendix II. This produced the following equation for the trade balance (as a percent of GDP, as are the other variables):

$$T_t = 0.23282 + 0.99867 T_{t-1} + \varepsilon_t \quad (6)$$

where $\sigma_\varepsilon = 0.73198$ is the standard deviation of shocks to the trade balance, calculated as a percent of GDP, so that the typical trade balance shock corresponds to roughly three-quarters of a percent of GDP. A crisis is triggered if the country's reserves would otherwise go below a

[8] Sections IV and V report on simulations at a monthly frequency, however.
[9] See Masson (1999a).

certain threshold, here assumed to be zero. A crisis is best interpreted as a (possibly partial) default[10] on contracted debt in a proportion $0 \leq \bar{\delta} \leq \infty$ that prevents reserves from going negative. It is assumed that $\bar{\delta} = 1$, so that a default reduces the value of debt by half.

We first summarize in Table 1 the properties of the **actual** data on emerging market spreads for the periods of data availability between 1994 and 2002, for the emerging market countries for which JP Morgan collected data (the countries and time periods are detailed in Masson (2003), which also presents a more detailed analysis). The daily and monthly (month-end to month-end) changes both exhibit a large amount of excess kurtosis in comparison to the normal (whose kurtosis is 3.0). Interestingly, the distribution is nearly symmetric, and the mean change is close to zero. As well as skewness and kurtosis, the table reports the Jarque-Bera test statistic of the null hypothesis of normality, based on those two moments[11], and the maximum and minimum change in the spread.

************* Table 1 about here *******************

The distribution of **simulated** daily changes in emerging market spreads over the US treasury bill rate is given in Table 2, for simulation runs of length 28000 days (of which the first 100 were dropped to minimize the effects of initial conditions). It can be seen that unlike the actual data for changes in spreads, the simulations do not produce fat tails. In fact, the distribution has thin tails, not fat tails, since the kurtosis is less than the value of 3 that characterizes the normal. As a result, normality is rejected at a very small p-value, using the Jarque-Bera test. It is also the case that the simulations tend to produce serial correlation in changes in spreads (not reported); since this stylized fact is easy to replicate, we do not dwell on it further.

******************** Table 2 about here ***************************

Table 2 explores whether the absence of fat tails is robust to changing the model's parameters. The table presents statistics for simulations with alternative values for risk aversion, the probability of experimentation, the probability of investing in a given period, the maximum value for the probability of default, the standard deviation of shocks to the trade balance, and the endogeneity of wealth. Increasing the value of $\pi^{\max}$ from 0.1 to 0.5 causes dramatic increases in the dispersion of changes in spreads, while introducing risk aversion also has that effect, but more moderately. In contrast, decreasing the probability of experimentation, not surprisingly, lowers volatility. Other changes reported in the table have relatively modest impacts. In particular, a striking result is that multiplying by 10 the standard deviation of shocks to the trade balance (row 6) scarcely affects the distribution of the change in spreads. Thus, the fluctuations in the model—capital flows into and out of emerging markets that provoke occasional crashes and spikes in spreads—result here from shifting expectations rather than economic fundamentals (see also Arifovic and Masson (2004)). Since investors do not in fact observe the fundamentals,

[10] However, it can also be interpreted as a devaluation that lowers the foreign currency value of debt contracted in domestic currency. See Masson (1999a).
[11] It is distributed as a chi-square with 2 degrees of freedom.

the changes in their expectations are solely the result of fitness comparisons and experimentation.

None of the above changes has a great effect on skewness or kurtosis, and the Jarque-Bera test strongly rejects normality, as before. Thus, though these changes affect the range and variance of the distribution of changes in spreads, in all cases the distribution does not exhibit fat tails. The model as it stands does not seem able to replicate this stylized fact, one that is strongly present in the actual data. Our results contrast with the view expressed by Cont and Bouchaud (2000) to the effect that herd behavior (which we have present in our model in the form of imitation) is sufficient to produce fat tails in returns.

IV. Simulating a Model with Several Emerging Market Bonds

We then proceed to simulate a model in which there is more than one emerging market bond, in order to consider contagion phenomena. It is an important stylized fact that actual returns on emerging market debt seem to be highly correlated—perhaps more so than would be dictated by economic fundamentals, and an extensive literature discusses why crises might occur together in several emerging market countries. In the data (see Masson (2003)), changes in emerging spreads are indeed highly correlated—though this may not be the same as the co-occurrence of crises[12]. This is a feature that we would hope our model with several emerging market countries could reproduce.

As detailed in Appendix I, in this case, investors need to formulate estimates of the covariance of defaults among emerging market countries when allocating their portfolios. We take the correlation between them (or correlation matrix, in the case of more than 2 risky assets), but not the variances, as given; that is, investors do not update their priors concerning the extent that returns move together. One would expect that the perceived correlation would be a key parameter for explaining the existence of contagion.

The simulations were performed with a model with only two emerging market countries, each of which with parameters identical to the one emerging market case described above, with uncorrelated shocks to the trade balance. Because the added complexity of the model slowed execution time and in order to approximate more closely the typical time horizons of investors, we simulated at a monthly, not daily frequency. Time aggregation will tend to reduce excess kurtosis, but in the actual data it is still present at a monthly frequency (see Table 1).

An interesting issue is whether it is *investors' beliefs* that emerging market bonds are similar that induces contemporaneous crises. A variant of this argument has been used to explain the East Asian crisis, namely the "wake up call hypothesis" (Goldstein, 1998): a crisis in one country (Thailand) made investors realize that there were fundamental problems in neighboring countries with similar institutions—or investors may have been misled into thinking there were, even though this was not the case. If the latter, true economic fundamentals might be uncorrelated, though investors treated them as being correlated. Investors might either over-

[12] In particular, our simulations of the market maker model described below produce high correlations of changes in spreads, but not the co-occurrence of defaults.

estimate the degree of correlation, or think that other investors' portfolio shifts would produce correlation where none existed. Such behavior might conceivably produce a self-fulfilling, rational expectations equilibrium, in which there was contagion across emerging market bonds.

Table 3 gives the effect of varying the perceived correlation in the defaults by the two banks[13]. This determines the expected degree of covariance of their returns; all investors assume the same (unchanging) correlation, though investors continue to formulate different expectations of the probability of default. While it is unrealistic to suppose investors all perceive the same correlation, this polar case is considered to see whether a case can be made for self-fulfiling correlations as a source of contagion. The table reports on simulations where that correlation is fixed at a value that goes from –1.0 to 1.0. To repeat, the covariance of the two countries' macro-economic fundamentals, in these simulations, is actually zero: innovations to the two emerging markets' trade accounts in their balance of payments are in fact uncorrelated.

**************** Table 3 about here ***************************

The striking result is that for most of the values of the perceived correlation coefficient– even when it is positive—the correlation of returns is negative. Thus, there seems to be no self-fulfilling element to expectations here: even if investors believe that crises will occur simultaneously in the two emerging markets, this does not provoke co-movement of their spreads relative to US Treasuries. Thus, this conjectured co-movement is not sufficient to explain any contagion phenomenon, quite to the contrary.

The explanation for the negative co-movement in spreads is simple: since the two assets are substitutes in an investor's portfolio, there is a tendency to increase holdings of one when the other asset is viewed as having a greater chance to default. This portfolio substitution effect dominates any effect resulting from treating the two assets as somehow members of the same risk class. Indeed, the effect of conjecturing a negative correlation among their returns makes investors want to hold both assets, since doing so reduces overall portfolio risk. The reason for this can be seen in equation (A4) of Appendix I. A more negative correlation, other things equal, increases demands for the two securities. Paradoxically, this may thus produce positive correlation in their defaults, not the negative one that was conjectured, since by varying holdings in tandem investors will provoke contemporaneous booms and crises. The converse applies when investors conjecture positive correlations. However, these effects again are very small on the joint distribution of emerging market spreads, producing only very slight differences in their correlations. Moreover, differences in the latter are not systematically related to the perceived correlations of default.[14]

In sum, the simple model with two emerging markets does not fit the stylized facts than one with one market, since in addition to producing thin tails (kurtosis is less than for the normal for each of the simulations in Table 3), it does not provide an explanation for contagion. Thus, the model either needs to be replaced or extended in order to provide an adequate depiction of

[13] We simulated 1000 months (corresponding roughly to the pooled sample of actual data), but dropped the first 100 observations.

[14] An alternative, which we have not tried, is to make investors' expectations of the probabilities of default in the two emerging markets move in tandem. This would, by construction, force their interest rates to move together.

regularities in emerging market debt. In the next section, we consider an extension that models the varying liquidity of the market through the introduction of a market maker who varies his bid-ask spread as a function of the market's volatility and his inventory position.

V. Endogenous Liquidity: Introducing a Secondary Market and a Market Maker

Conversations with market professionals suggest that emerging market bonds suffer from periods of pervasive illiquidity, and that this applies across a range of countries rather than being localized in just a single country facing difficulties in its balance of payments[15]. We therefore expand the model by introducing a secondary market in emerging market debt, bonds now assumed to have two periods to maturity (except the US asset, which takes the form of cash). Investors purchase debt from emerging market governments when they are issued (the primary market), with rates of interest determined as described above. However, if investors want to sell before maturity, or buy a bond with only one period remaining until maturity, they need to deal in the secondary market with a market maker who quotes a buy and a sell price, which differ by the bid-ask spread, and whose average price reflects the size of the market maker's inventory of the security.

There is an extensive literature modeling the behavior of market makers.[16] Market makers are usually assumed to avoid taking speculative positions, making their income by trading, not investing. Therefore, the size of their inventory of securities has an important role in influencing the size of their bid-ask spread (O'Hara and Oldfield, 1986). Shen and Starr (2000) develop a model of optimal market maker behavior in which the spread depends positively on the security price's volatility, the volatility of order flow, and the market maker's net inventory position. We follow them in making liquidity (which is inversely related to the size of bid-ask spreads) depend on the market maker's costs, which rise with increasing volatility. We extend their model by including more than one security. We also assume (as do Shen and Starr) that threat of entry leads to zero profits in the long run. Thus, the model mimics competitive behavior (though for simplicity we include only a single market maker in the model). However, the Shen-Starr model takes the evolution of prices over time as exogenous. This is not useful for our purposes, since market makers deal sequentially with many investors and could acquire unbounded inventory positions unless they adjusted the price. Instead, market makers in our model make decisions both on the price level and on the bid-ask spread; we assume that these decisions are separable.

We proceed to describe the investor's decision tree and the role of the market maker.
Investor. Instead of 3 assets—the riskless US bond and 2 emerging market bonds—we now have 5, since investors hold emerging market bonds with remaining terms to maturity of 1 and 2

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[15] One definition of liquidity is the narrowness of the bid-ask spread; another is based on the volume of transactions (see Duffie and Singleton (2003)). Unfortunately, data for neither measure are publicly available on a consistent basis for emerging market bonds. In future work, we hope to obtain proprietary data from market participants on spreads, as well as quantitative data on the volume of transactions.

[16] LeBaron (2001) discusses alternative market structures for agent-based financial models, including models with market makers, while Farmer and Joshi (2002) study the interaction of investors with different trading strategies in a model with market makers.

periods. However, the two maturities of bond issued by a given country are viewed as perfect substitutes, since the probability of default is the same in both periods and the investor does not face stochastic consumption shocks so is indifferent to the term to maturity[17]. Thus, at the beginning of each period, each investor calculates optimal holdings (x_0, x_1, x_2) of US, emerging market 1, and emerging market 2 bonds, respectively, as described in Appendix I. After having redeemed maturing bonds, she has holdings (ω_1, ω_2) of emerging market bonds which she compares with her desired holdings. Let $\Delta_j = x_j - \omega_j$ (we omit here the index *i* that characterizes the investor).

If $\mathbf{\Delta_j > 0}$, she buys $\mathbf{\Delta_j}$ one-period bonds from the market maker or new two-period bonds directly from the emerging market country, depending on whether or not

$$\frac{(1+r_{j,-1})^2}{p_j^b} > 1 + r_j$$

That is, the investor chooses the bond with the highest return (since bonds pay 2 periods' interest at maturity, the market maker's selling price on a one-period bond p_j^b has to reflect accrued interest).

If $\mathbf{\Delta_j < 0}$, then the investor sells $\mathbf{-\Delta_j}$ to the market maker, unless

$$\frac{(1+r_{j,-1})^2}{p_j^s} < 1 + r^*$$

That is, the investor liquidates her excess holdings unless the market maker's price is so low that the return to holding on to them is greater than the return from investing in the safe asset. Since the investor may be risk averse and demand a preimum for holding risky assets (which is already embodied in her desired allocation between the safe asset and the 2 risky asset classes), this rule of thumb sets a conservative lower bound to the price an investor will accept.

Market maker. The market maker is assumed to be a financial intermediary who covers costs but takes no speculative position; he aims to minimize his exposure, long or short. Assuming that there are the same quadratic costs to deviating from zero holdings in either bond (as in Shen and Starr), his bid-ask spread could depend on the volatilities in the two markets. We parameterize the weight given to the own-volatility and the other bond's volatility using the ξ parameter; we allow for various possibilities. In the base-case simulations reported below, we assume equal weights on the two volatilities, but as we shall see below allowing **no** volatility spillovers produces similar simulation results.

We calculate volatilities as exponentially-weighted averages of the absolute value of the change in the rate in the primary market; a fixed window with equal weights would be an alternative, but one which requires storing a larger volume of past data. The constant *A* is chosen to roughly enforce the zero long-run profit constraint, as would occur under competition among market makers, even though for convenience we model only a single market maker; any

[17] However, the actual occurrence of default may differ since if reserves go to zero, the borrower defaults on bonds that mature that period, while it may (if reserves are positive by then) not default on second period bonds.

excess profits are remitted to the emerging market governments in equal shares (and hence augment their reserves). Thus, the market maker could be thought of as an agent of the emerging market governments. The bid-ask spread is set at the beginning of each period. Within each period, the market maker deals sequentially with investors who want to transact, quoting a buy or a sell price. The market maker (since he does not initially hold an inventory of bonds) is not prevented from going short, but adjusts his price as he transacts in order not to accumulate too large long or short positions. Thus,

$$spread_t = A + B \bullet [(1-\xi) \bullet vol_{1t} + \xi \bullet vol_{2t}]$$
$$vol_{jt} = (1-\gamma)\sum_{k=1}^{\infty} \gamma^{k-1} |\Delta r_{jt-k}|$$
$$price_{jt} = (1 + r_{jt-1})e^{-\theta X_{jt}}$$

where A,B, ξ, γ, and θ are positive parameters, vol_{jt} is the volatility of rate j, Δr_{jt} is the one-period change in the primary market interest rate set on bond j, and X_{jt} is the market maker's net holdings of security j. In the limit $\gamma \rightarrow 0$ volatility depends only on the absolute value of the change in the rate in the most recent period; $\gamma = 1$ weights all past periods the same. When $\xi=0$, only the security's own volatility influences the spread; when $\xi=1/2$ (the benchmark case), the two securities' volatilities have an equal effect.

We also study the cases where liquidity is not an issue, i.e. spreads are zero; and where on the contrary there is no liquidity in the secondary market, that is investors have to hold their bonds for the full two periods until maturity. These two cases are nested in the model: in the first case, by $A=B=0$; in no liquidity case, by A large enough that investors choose never to transact. In the first case we also increase the value of θ and term it "market clearing," even though because of the sequential nature of the market makers' transactions he does not play the role of a Walrasian auctioneer who first polls all investors for their excess demand schedules and then finds a market clearing price.

The spread is fixed at the beginning of the period, while the market maker's price varies within the period, depending at each point on the inventory position that has resulted in his transactions with investors. Thus he buys at p^b_{jt} and sells at p^s_{jt}, where the prices are given by

$$p^b_{jt} = price_{jt} * (1 - spread_t)$$
$$p^s_{jt} = price_{jt} * (1 + spread_t)$$

The market maker's holdings evolve as he transacts with each investor. After dealing with investor i,

$$X^{(i)}_{jt} = X^{(i-1)}_{jt} - \Delta^{(i)}_{jt} \bullet price^{(i)}_{jt} \bullet (1 + sign(\Delta^{(i)}_{jt}) \bullet spread_t)$$

The latter term depends on whether the transaction is a buy or a sell, that is, on the sign of the investor's excess demand. In fact, the simulations randomize the order investors transact with the market maker so that investors *i-1, i, i+1,…* vary from period to period. At the end of each period of buying and selling with all the investors who want to transact, the market maker either covers his short position by buying one-period bonds from the emerging market country, or holds his long position until it matures in the following period.

First, we consider the two polar cases, perfect **market clearing** (zero bid-ask spreads) and **no liquidity** (infinite bid-ask spreads), as well as an intermediate parameterization with **endogenous liquidity** in which investors can, for a price, transact in the secondary market. Summary statistics for both primary and secondary markets—that is the interest rate on new issues and their equivalent on one-period-old bonds that correspond to transactions with the market maker--are reported in Table 4, where all the simulations were run over 1800 periods (months). Across cases, the same seed was used, so that the results are comparable and are not due to the random numbers chosen[18]. The first 500 simulated periods were ignored in each case—a longer period than before because this model seemed more sensitive to initial conditions.

************************Table 4 about here ******************************

There is a stark difference in the results for the three cases. The "no liquidity" case (the left panel) gives tails which are only slightly greater than the normal: kurtosis is around 4, compared to 3 for a normal distribution. Changes in spreads for the two emerging market countries are negatively correlated, as in the results described above. In contrast, both the "market clearing" case and the intermediate "endogenous liquidity" case, in which liquidity is provided by the market maker but varies endogenously depending on historical volatility, produce considerably fatter tails than the normal[19]. Though it would be expected that the secondary market would exhibit the larger kurtosis, excess kurtosis applies to **both** the primary and secondary markets, though the determination of interest rates in the primary market has formally not been changed. Instead, it seems that transactions in the secondary market, by affecting the amounts held of primary securities, increase the likelihood of large changes in primary interest rates. Though not reported, it is the case that there is always positive correlation, for a given emerging market country, between the change in spreads in primary and secondary markets. It is even true in the "market clearing" case that for one of the countries, kurtosis is greater in the primary than the secondary market, but this seems to be an artefact due to the specific seed used. The market clearing case also exhibits great instability of interest rates, as shown by the very large standard deviations and ranges of fluctuation in secondary markets. Finally, the correlation between the two countries' changes in spreads, instead of negative (the "no liquidity" case) is now strongly positive.

In order to elucidate the factors contributing to the excess kurtosis and positive correlation, we simulate the model with different parameters for market maker and investor behavior, with the results reported in Table 5 (the values taken by these parameters in Table 4 are given in brackets). We first explore whether making spreads depend on the other country's

[18] In addition, here the simulations for simplicity impose a zero US interest rate and a zero rate of consumption on the part of investors; the trade balances for both emerging market countries are also zero. As a result, the sum of the net worth of emerging market governments and investors is a constant (market makers' net worth is also constant, since they remit profits to governments).

[19] All of the market maker simulations use a value of $\theta=0.0001$. The market clearing case increases this parameter by an order of magnitude, to $\theta=0.001$.

volatility ($\xi>0$) is crucial to the positive correlation discovered in Table 4. The first panel reports results with $\xi=0$. It is still the case that there is positive correlation in changes in the two emerging market spreads. Thus, an active secondary market is a feature that contributes to the observed co-movement in emerging market interest rates, whether there are "volatility spillovers" or not. In these simulations, the economic fundamentals are non-stochastic (and uncorrelated), so there is no reason for co-movement in changes in their interest rates. As we saw above, investors' portfolio behavior was insufficient to produce co-movement (and in fact produced negative correlation), but the introduction of a liquid secondary market is able to do so.

************************** Table 5 about here ***************************

In addition, we revisit the issue of how investors' expectations formation might influence the results, by varying key parameters relative to the basic "market maker" case. To summarize the formation of expectations of the probability of default, each investor either compares her beliefs to those of one or several randomly-chosen comparators (2 in Table 4)--where an investor is more likely to be chosen as a comparator the higher is his fitness--or, with probability p_{ex} *(0.05)* experiments by choosing a new strategy chosen randomly. Fitness is calculated over a particular horizon for past returns, which we call "investor memory"; in the simulations of Table 4, investor memory is set at 2 months. Expectations of the default probability are assumed bounded; that bound is the "maximum probability of default" (0.2), labelled π^{max} in earlier simulations.

In Table 5, we see if changing each of these parameters in turn significantly affects the distribution of returns; the other parameters in each case are set to their values in the "endogenous liquidity" case of Table 4[20]. We summarize the results as follows. **First,** the number of comparators matters for kurtosis but not in a monotonic way. It seems that having both fewer (1) and more (5) comparators (relative to 2) increases the possibility of large and sudden shifts of opinion, producing occasional, larger interest rate movements. **Second,** longer investor memory (12 rather than 2 months) also increases kurtosis, but reverses the positive correlation. **Third,** increasing the upper bound on the expected probability of default π^{max} (to 0.5) increases the variance of the distribution (not surprisingly, since now expectations take values over a wider range) but does not significantly affect kurtosis. More surprisingly, however, the correlation properties of the changes in the two emerging market countries' returns change from positive to negative. It appears that allowing for higher interest rates (associated with greater expected probability of default) qualitatively affects outcomes. **Fourth**, increasing the probability of experimentation (to 0.333) substantially **reduces** kurtosis in the primary market. In this case, there is not enough imitation causing the herd behavior that is an important source of financial crises (and large changes in interest rates). Also interesting is that more experimentation **reverses** the positive correlation that emerged in the market maker model.

Despite its apparent simplicity, the model is complicated enough that changes in parameters produce non-trivial consequences that cannot be easily be inferred. Clearly, further

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[20] These simulations are done using the same random seed as in Table 4, but in order to examine sensitivity to random draws we redid simulations with 10 different realizations of the random variables. These simulations are available from the authors.

investigation is needed to map out the regions in the parameter space where the phenomena of interest, excess kurtosis and positive correlation of changes in emerging market returns, occur. It is also true that other information structures are possible and relevant (see, e.g., Watts 1999), and are worth exploring in the context of our model. For instance, there could be a group of trend setters (e.g. Goldman Sachs, Tiger Fund, or other large investment banks or hedge funds) whose strategies are widely watched and imitated. Or it could be that imitation is regional, with traders in New York, London, and Tokyo constituting separate groups. Exploring these possible networks among traders may be a subject of our future research, which we would hope to make precise by interviews with actual market participants. It may also be worth considering different access to information; market makers (and big investment banks generally, whether or not they make a market in a particular security) are widely believed to benefit from superior knowledge relative to other investors (in part because of their contacts with emerging market countries and their role as underwriters). Agent-based models are particularly useful for studying such interactions (see, for instance, Epstein and Axtell (1996)).

We would nevertheless conclude on the basis of our preliminary results that the introduction of a market maker and changing liquidity makes a significant step toward reproducing the stylized facts describing the actual data for emerging market spreads. There are two crucial linkages here that help to reproduce some of the features of the actual data on emerging market spreads, and would be present in any reasonable model of liquidity provision and portfolio selection. The **first** linkage results from the assumption that the market maker's spread increases with volatility. However, having the market maker respond to volatility in one market by raising bid-ask spreads in the other market as well is not a necessary feature to produce the co-movements that we see in the simulations—similar properties emerge when $\xi=0$. **Second**, investors choose to retain their holdings in emerging market debt when liquidity is too low, that is, the prices at which they can sell are too unattractive (relative to holding on to their bonds). In any model of optimal portfolio selection, there will be prices at which investors will refuse to transact in the secondary market, and hence their initial holdings will matter. When expectations shift suddenly in the same direction, low liquidity will mean that prices need to adjust a lot in order to make it possible for investors to trade and get closer to their desired portfolio positions. Being locked into their holdings may cause them to incur large losses if there is a subsequent default, and this will then change the dynamic of their expectations formation and their desired bond holdings next period. The endogeneity of spreads is also important in causing serially correlated effects. Iori (2002) finds that if thresholds defining a no-trading range are constant over time (or zero), then volatility clustering does not occur in her model.

VI. Concluding Comments

We have identified various important features of the data for interest rates on emerging market debt, and formulated a model, which, after being extended to include endogenous liquidity, is able to replicate some of those features--in particular, fat tails in the distribution of the changes in spreads (against US treasury securities) and positive correlation between changes in emerging market interest rates (even though the economic fundamentals are assumed uncorrelated in the simulation model). The analysis is necessarily exploratory and suggestive, rather than definitive. Nothing proves that some other model, even a reasonably parsimonious one like that presented here, may not replicate the stylized facts as well as, or better than, this one[21]. And despite its simplicity, there needs to be more exploration of the effects of changing parameters or structure in order to understand the essential factors at work. We have identified the networks involved in imitation as a promising avenue to explore. Our model, like other models of heterogeneous agents, has interesting interactions between cross sectional variability and time series volatility. Moreover, frictions that inhibit continuous rearranging of portfolios produce non-convexities with interesting distributional implications.[22] We hope to understand better in future work the key aspects of heterogeneity that produced the observed time series properties.

Other financial models have also produced returns with fat tails, for instance through assuming that there are two types of traders, noise traders and fundamentalists, whose numbers vary in some fashion (see for instance, Lux (1998) and Lux and Marchesi (1999)). Most of the analysis to date has been applied to equity, foreign exchange, or developed country bond markets, all of which are deep, liquid markets. Studying the emerging debt markets is valuable in itself, not least because sharp movements in rates are associated with fears of (and the occurrence of) large defaults and currency devaluations. But, in addition, periods of illiquidity in these markets are much more of an issue than for advanced country financial markets, and contagion across markets has generated more concern also. Indeed, Rigobon (2002) finds that the upgrade of Mexico by rating agencies increased the universe of potential investors, and significantly decreased co-movements with other emerging markets. He therefore ascribes an important role to liquidity factors in explaining contagion. We hope to obtain data from market participants that confirms the link between volatility and illiquidity. In any case, the results of our paper suggest that endogenous liquidity can, in and of itself, produce some of these features, and hence warrants further research in the context of emerging market fluctuations.

[21] For example, Ben Klemens (personal communication, November 3, 2003) has produced excess kurtosis in the distribution of spreads by modifying this model such that each investor revises her expectations of default in light of market consensus as revealed in the secondary market price. The rationale for such a model is described in Klemens (2003).

[22] There is an analogy here with the literature on income inequality and growth (see Galor and Zeira (1993)).

Appendix I. Investor Portfolio Selection with Risk Aversion

We consider here the case with one riskless asset and two risky assets; it generalizes easily to the case of several risky assets. For notational convenience, we ignore time subscripts which apply to all rates of return, asset proportions, and default probabilities. The return R^i on investor *i*'s portfolio is given by

$$R^i = x_0^i r^* + x_1^i (\frac{1+r_1}{1+\delta_1} - 1) + x_2^i (\frac{1+r_2}{1+\delta_2} - 1) \tag{A1}$$

where r^*, r_1, and r_2 are the interest rates on the riskless (US) asset, and on emerging market bonds 1 and 2, respectively. Realized default proportions are denoted δ_1, δ_2 on assets 1,2, respectively. Portfolio proportions x_j^i sum to 1 for each investor *i*, so we can also write the portfolio return as

$$R^i = r^* + x_1^i (\frac{1+r_1}{1+\delta_1} - 1 - r^*) + x_2^i (\frac{1+r_2}{1+\delta_2} - 1 - r^*) \tag{A2}$$

By assumption, the investor maximizes a utility function $U^i(R^i)$ with respect to x_1^i, x_2^i, given her expectations of default $\delta_j^i = E^i(\delta_j)$. Investors also differ in their degree of risk aversion (denoted $1/b^i$ below). The first order conditions are:

$$E^i\left[U'^i(R)\left(\frac{1+r_1}{1+\delta_1} - 1 - r^*\right)\right] = 0$$

$$E^i\left[U'^i(R)\left(\frac{1+r_2}{1+\delta_2} - 1 - r^*\right)\right] = 0$$

We expand U^i in a second-order Taylor's expansion around the expected return

$$\overline{R}^i \equiv E^i(R^i) = r^* + x_1^i(\mu_1 - r^*) + x_2^i(\mu_2 - r^*)$$

where $\mu_j^i \equiv E^i(\frac{1+r_j}{1+\delta_j} - 1)$:

$$\begin{aligned} U'^i(\overline{R}^i)(\mu_1^i - r^*) + U''^i(\overline{R^i})(x_1^i \operatorname{var}_1^i + x_2^i \operatorname{cov}_{12}^i) = 0 \\ U'^i(\overline{R^i})(\mu_2^i - r^*) + U''^i(\overline{R^i})(x_1^i \operatorname{cov}_{12}^i + x_2^i \operatorname{var}_2^i) = 0 \end{aligned} \tag{A3}$$

where

$$\operatorname{var}_j^i \equiv E^i(\frac{1+r_j}{1+\delta_j} - \mu_j^i)^2$$

$$\operatorname{cov}_{12}^i \equiv E^i[(\frac{1+r_1}{1+\delta_1} - \mu_1^i)(\frac{1+r_2}{1+\delta_2} - \mu_2^i)]$$

Writing

$V^i \equiv \begin{bmatrix} \text{var}_1^i & \text{cov}_{12}^i \\ \text{cov}_{12}^i & \text{var}_2^i \end{bmatrix}, x^i \equiv \begin{bmatrix} x_1^i \\ x_2^i \end{bmatrix}, \mu^i \equiv \begin{bmatrix} \mu_1^i \\ \mu_2^i \end{bmatrix}$ and letting $b^i \equiv -\frac{U'^i(\bar{R}^i)}{U''^i(\bar{R}^i)}$ the first order conditions can be solved to yield

$$x^i = b^i V^{i^{-1}}(\mu^i - r^*) \qquad \text{(A4).}$$

As is well known from portfolio theory, the composition of the risky asset portfolio will not depend on the degree of risk aversion (the inverse of b^i)—that is, the portfolio proportions captured by ratio x_1^i / x_2^i will be independent of b^i. However, the proportion of the total portfolio held in the safe asset will depend inversely on b^i. In the limit as $b^i \to 0, x^i \to 0$ and all wealth is held in the safe asset. In the simulations, b^i varies across investors, and is initialized by drawing from a uniform distribution in a pre-specified range.

The expected return on each bond depends on the expected default. Default size is assumed known and exogenous (and $\bar{\delta} = 1$, which implies that default reduces the value of the bond by half), so the key variable is each investor's estimate of the probability of default on bond bond *j*, π_j^i, which varies from investor to investor. So $E^i(\delta_j) = \pi_j^i \bar{\delta}$, and the expected return for a given investor can be written:

$$\mu_j^i = \pi_j^i \left(\frac{1+r_j}{1+\bar{\delta}} - 1 \right) + (1-\pi_j^i) r_j = r_j - \frac{1+r_j}{1+\bar{\delta}} \pi_j^i \bar{\delta},$$

its variance

$$\text{var}_j^i = E^i \left(\frac{1+r_j}{1+\delta_j} - 1 - \mu_j^i \right)^2 = \pi_j^i \left(\frac{1+r_j}{1+\bar{\delta}} - 1 - \mu_j^i \right)^2 + (1-\pi_j^i)(r_j - \mu_j^i)^2,$$

and the covariance between bonds *1* and *2*

$$\text{cov}_{12}^i = E^i \left[\left(\frac{1+r_1}{1+\delta_1} - 1 - \mu_1^i \right) \left(\frac{1+r_2}{1+\delta_2} - 1 - \mu_2^i \right) \right] = \rho_{12}^i \sqrt{\text{var}_1^i \, \text{var}_2^i},$$

where ρ_{12}^i is the investor *i*'s estimate of the correlation between the two asset's returns. In the simulations this correlation is a fixed parameter which is assumed to describe the expectations of all investors.

Appendix II. Currency Crash Models at Different Frequencies

Instead of (1) the flows T_t need to be divided by *n*, where *n* is either 12 (monthly) or 365 (daily):

$$R_t = R_{t-1} + D_t - (1+r_{t-1})D_{t-1} + T_t / n \qquad (1')$$

All interest rates need to be converted from annual to a monthly or daily frequency:

$$(1 + r_t^{(n)}) = (1 + r_t)^{1/n}$$

where r_t on the RHS stands for the annual data, $r_t^{(n)}$ on the LHS stands for the monthly or daily data. As for the probability of a default, let π_t^i be the probability over the coming year, and ${}^{(n)}\pi_t^i$ for a fraction *1/n* of the year. Assuming that the probability of a default in each of the months or days is independent of the other, then

$$1 + {}^{(n)}\pi_t^i = (1 + \pi_t^i)^{1/n}$$

Another problem is the trade balance equation, however, because the lagged endogenous variable now refers to the previous month's value, not the previous years. Persistence should be greater at higher frequency, and so should be the coefficient on the lagged endogenous variable. Suppose that the true adjustment takes place at the higher frequency, as in the following equation

$$T_t = a + bT_{t-1} + \varepsilon_t \qquad (4')$$

If we go to a lower frequency, e.g. the time period is twice as long,

$$\begin{aligned} T_t &= a + b(a + bT_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\ &= a(1+b) + b^2 T_{t-2} + b\varepsilon_{t-1} + \varepsilon_t \end{aligned}$$

More generally, if the time period is n times as long as the initial (high frequency) data, then

$$\begin{aligned} T_t &= a(1 + b + \ldots + b^{n-1}) + b^n T_{t-n} + \varepsilon_t + b\varepsilon_{t-1} + \ldots + b^{n-1}\varepsilon_{t-n+1} \\ &= a\frac{1-b^n}{1-b} + b^n T_{t-n} + \varepsilon_t + b\varepsilon_{t-1} + \ldots + b^{n-1}\varepsilon_{t-n+1} \end{aligned}$$

It is this equation that is assumed to have resulted in the coefficient estimates from annual data, say

$$T_s = \alpha + \beta T_{s-1} + u_s$$

where *s* is indexed by years. If we infer back from the lower frequency data, say equation (4) above on annual data, we can calculate what the coefficients in (4') should be from:

$$a\frac{1-b^n}{1-b}=\alpha \qquad\qquad b^n=\beta$$

Similarly, the variance of the shocks to the trade balance needs to be adjusted downward using

$$\sigma_\varepsilon^2=\frac{1-b}{1-b^n}\sigma_u^2$$

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Table 1. Summary Statistics for the Distribution of Actual Changes in Emerging Market Spreads, All Countries[23]

(spread data in percentage points)

	daily	monthly
Mean	0. 00214	-0.0219
Standard Deviation	0.4832	0.5585
Skewness	-0.305	-5.70
Kurtosis	86.06	82.34
Largest	11.54	4.09
Smallest	-10.70	-8.46
Jarque-Bera test for normality	8,004,456	347,207
Number of Observations	27,842	1,297

[23] Countries are:Argentina, Bulgaria, Brazil, Colombia, Ecuador, Korea, Morocco, Mexico, Nigeria, Panama, Peru, Philippines, Poland, Qatar, Russia, Turkey, Ukraine, Venezuela, and South Africa. Daily data spanned December 31, 1993 to July 19, 2002, or shorter periods when a country's data was not available for the whole period.

Table 2. One Emerging Monthly Model: Effects of Daily Changes in Parameters on the Distribution of Changes in Emerging Market Spreads, in Percentage Points (27, 900 Observations)

Simulation	Parameter Values							Distribution Statistics					
	Mem. Length	Pex	Pinv	π^{max}	b^{max}	$\sigma\varepsilon$	Wealth Endogenous	Range	Mean 1/	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
(1)	1	0.333	0.9	0.1	∞	0.73	N	(-1.26, 1.59)	.0004	0.364	0.049	-0.119	11,316
(2)	1	0.333	0.9	0.1	∞	0.73	N	(-1.25, 1.52)	.0006	0.366	0.045	-0.117	11,300
(3)	5	0.333	0.9	0.1	∞	0.73	N	(-1.25, 1.45)	.0005	0.348	0.016	-0.053	10,834
(4)	5	0.333	0.9	0.5	∞	0.73	N	(-7.86, 6.68)	-.0020	1.731	0.018	-0.031	10,683
(5)	5	0.333	0.9	0.5	5	0.73	N	(-6.32, 6.62)	.0060	1.539	0.052	0.032	10,252
(6)	5	0.333	0.9	0.5	5	7.32	Y	(-6.37, 7.18)	-.0015	1.537	0.070	0.033	10,253
(7)	5	0.167	0.9	0.5	5	0.73	Y	(-5.96, 5.42)	-.0005	1.328	0.005	0.042	10,170
(8)	5	0.167	0.5	0.1	5	0.73	Y	(-7.10, 6.42)	.0081	1.520	0.067	0.176	9,294
(9)	5	0.167	0.5	0.1	1	0.73	Y	(-8.07, 7.67)	.0119	1.723	0.029	0.081	9,909
(10)	5	0.167	0.5	0.1	100	0.73	Y	(-6.24, 6.97)	.0048	1.426	0.145	0.179	9,348

1/ multiplied by 10000.

Table 3. Two Emerging Markets: Simulated Distribution of Monthly Changes in Emerging Market Spreads, in Percentage Points (900 Observations)

Perceived correlation of defaults	-1.0	-0.5	0.0	0.5	1.0
Emerging Market 1					
Mean	-0.0004	-0.0003	-0.0002	-0.0002	-0.0004
Std. deviation	1.893	1.648	1.691	1.664	1.642
Skewness	-0.0000	-0.0038	0.0208	0.0119	0.0349
Kurtosis	0.2133	0.0954	0.0978	0.0851	0.1571
Emerging Market 2					
Mean	-0.0002	0.0001	-0.0000	-0.0002	-0.0000
Std. deviation	1.898	1.650	1.679	1.652	1.642
Skewness	-0.0110	0.0462	0.0366	0.0048	0.0291
Kurtosis	0.3432	0.1458	0.0856	0.0758	0.1409
Correlation of simulated Changes in spreads	-0.0083	-0.0833	-0.0929	-0.1021	0.0066

Table 4. Effect of Endogenous Liquidity vs No Liquidity and Market Clearing: Summary Statistics for Monthly Changes in Emerging Market Spreads, Primary and Secondary Markets, in Percentage Points

	No Liquidity				Market Clearing				Endogenous Liquidity			
	secondary mkt		primary mkt		secondary mkt		primary mkt		secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2	Country 1	Country 2
mean	n.a.	n.a.	0.002	-0.002	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	-0.001
std dev	n.a.	n.a.	1.456	1.517	6.631	14.506	0.543	0.646	1.047	1.195	0.449	0.445
skewness	n.a.	n.a.	0.380	0.630	0.103	0.100	0.014	6.204	-0.009	0.264	-0.478	-0.442
kurtosis	n.a.	n.a.	3.983	4.085	17.078	12.119	15.068	133.242	4.758	13.476	3.233	4.200
J-B	n.a.	n.a.	84	149	10729	4503	7882	926456	167	5955	52	120
largest	n.a.	n.a.	8.02	8.84	48.27	96.53	5.40	13.13	4.35	10.08	2.09	2.31
smallest	n.a.	n.a.	-6.07	-8.40	-47.04	-95.21	-4.56	-3.58	-4.79	-7.91	-2.72	-3.03
correlation	n.a.		-0.423		0.430		0.565		0.121		0.592	

Table 5. Effect of Key Parameters on Simulations with Market Maker:
Summary Statistics for Monthly Changes in Emerging Market Spreads, in Percentage Points

Zeta=0 (0.5)

	secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2
mean	0.001	0.002	0.000	0.000
std dev	0.922	2.071	0.603	0.669
skewness	0.377	0.498	0.641	0.015
kurtosis	5.315	11.070	16.471	9.748
J-B	321	3579	9910	2465
largest	5.11	17.67	5.32	5.42
smallest	-4.55	-12.31	-4.73	-3.94
correlation	0.149		0.039	

Number of comparators=5 (2)

	secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2
mean	0.008	0.008	0.008	0.008
std dev	1.479	2.527	0.913	0.908
skewness	0.753	0.025	0.549	0.588
kurtosis	8.651	20.448	10.667	16.245
J-B	1851	16478	3247	9570
largest	10.29	23.04	6.69	7.19
smallest	-8.14	-23.68	-6.00	-7.08
correlation	0.132		0.276	

Number of comparators=1 (2)

	secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2
mean	0.000	0.000	0.000	0.000
std dev	0.627	0.962	0.433	0.397
skewness	1.267	0.302	2.150	1.003
kurtosis	40.639	17.111	32.780	11.962
J-B	77025	10797	49003	4565
largest	8.09	7.64	6.05	4.10
smallest	-7.04	-6.91	-2.48	-1.60
correlation	0.232		0.585	

Investor memory=12 months (2)

	secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2
std dev	3.514	3.711	1.095	0.955
mean	-0.002	0.002	0.001	0.003
kurtosis	52.337	33.515	10.243	7.839
skewness	0.077	0.047	1.231	0.608
J-B	131750	50398	3168	1348
largest	36.96	42.67	8.21	6.34
smallest	-38.99	-42.16	-6.32	-4.83
correlation	-0.081		-0.255	

Max probability of default=0.5 (0.2)

	secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2
std dev	7.672	7.300	3.687	3.833
mean	-0.001	-0.011	0.004	-0.007
kurtosis	19.722	14.664	9.170	4.794
skewness	-0.098	-0.010	1.190	0.699
J-B	15136	7364	2367	280
largest	59.63	56.98	27.23	23.76
smallest	-69.45	-61.29	-18.37	-18.37
correlation	-0.130		-0.119	

Probability of experimentation=0.333 (0.05)

	secondary mkt		primary mkt	
	Country 1	Country 2	Country 1	Country 2
std dev	8.923	7.478	1.511	1.483
mean	0.001	0.001	0.002	-0.001
kurtosis	5.454	8.889	2.013	1.107
skewness	0.030	-0.032	0.796	0.732
J-B	326	1877	190	310
largest	42.52	41.14	6.98	6.08
smallest	-44.72	-41.45	-4.76	-3.85
correlation	-0.194		-0.582	